Supplemental Online Appendix to "Using Regression Discontinuity to Uncover the Personal Incumbency Advantage"

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Contents

1 Overview

This supplemental appendix to the paper "Using Regression Discontinuity to Uncover the Personal Incumbency Advantage" is intended for online publication only. In Section 2, we show how the model in the main body of the paper, which was developed only for open seats at t where no incumbent retires at $t+1$, can be generalized to cases where some incumbents retire at $t+1$ and cases where the quality differentials are of arbitrary signs. Next, in Section 3, we show that our model can be written in terms of potential outcomes, and discuss how our double-counting result can be obtained directly from a potential outcomes based model.

2 Generalizing the Model

In this subsection, we show how the model for where no incumbent is running at t ("open seats at t") presented in the main body of the paper can be generalized to all kinds of seats. In doing this generalization, it becomes clear that the double-counting phenomenon is not exclusive to open seats at t, but also occurs in seats where an incumbent is running at t. As we show, however, in this latter set of seats recovering the personal incumbency advantage from the RD effect is complicated by the fact that, unlike in our open seats model where we invoke an incumbency-induced scare-off effect, the sign of the quality differential cannot be determined.

We model district's i Democratic Vote share at election $t+1$,v_{it+1}, as follows

$$v_{it+1} = Par_{it+1} + Z_{it+1}\theta + Z_{it+1}(D_{it+1} - R_{it+1}) + e_{it+1}$$

where the different terms are described below:

- Par_{it+1} is the district's Par at $t+1$: the baseline vote for a party in a district, given district's partisanship, election year's partisan trend, no incumbent candidate, and Democratic and Republican candidates of average quality.

- D_{it+1} and R_{it+1} are the added quality of the Democratic and Republican candidates, respectively, running at $t+1$ in district i, above the quality of the average open seat candidate in their respective parties as measured by Par (so $D_{it+1} = 0$ and $R_{it+1} = 0$ by construction in an open seat – i.e., open seat candidates are of average quality, which is measured in Par).

- As a result, $(D_{it+1} - R_{it+1})$ is the *quality differential* between Democratic and Republican candidates running at $t+1$

- $Z_{it+1} = 1$ if Democratic incumbent, $Z_{it+1} = 0$ if open seat, $Z_{it+1} = -1$ if Republican incumbent

- $I_{it+1} = 1$ if an incumbent (of any party) is running at $t+1$ and zero if the $t+1$ election is an open seat

- θ: personal *direct* incumbency advantage

- e_{it+1}: error term

Although not entirely evident in the equation above, v_{it+1} depends on v_{it} through Z_{it}. To see this more clearly, decompose Z_{it+1} as

$$Z_{it} = \theta \cdot \text{sign}\left\{v_{it} - \frac{1}{2}\right\}$$

where sign· denotes the sign function (sign $\{A\} = 1$ if $A > 0$, sign $\{A\} = -1$ if $A < 0$).

Taking the expectation of v_{it+1} conditional on $v_{it} = v$, we obtain:

$$\begin{aligned} E[v_{it+1}|v_{it} = v] =& E[Par_{it+1}|v_{it} = v] + \theta E[I_{it+1} \text{ sign}\{v_{it} - \frac{1}{2}\}|v_{it} = v] \\ &+ E[I_{it+1}(D_{it+1} - R_{it+1})|v_{it} = v] \end{aligned}$$

We define the right and left limits of the expectation of v_{it+1} conditional on $v_{it} = v$:

$$v^w_{t+1} = \lim_{v \to \frac{1}{2}^+} E(v_{t+1} | v_{it} = v)$$

$$v^l_{t+1} = \lim_{v \to \frac{1}{2}^-} E(v_{t+1} | v_{it} = v)$$

We also use the superscript w and l to denote the right and left limits of the expectation of any of the other random variables in the model conditional on $v_{it} = v$. Intuitively, we think of v^w_{t+1} as the average of v_{t+1} among barely-winner districts and of v^l_{t+1} as the average of v_{t+1} among barely-loser districts.[1]

Taking limits and expectation, the expressions for v^w_{t+1} and v^l_{t+1} are

$$v^w_{t+1} = Par^w_{t+1} + \theta I^w_{t+1} + I^w_{t+1}(D^w_{t+1} - R^w_{t+1})$$
$$v^l_{t+1} = Par^l_{t+1} - \theta I^l_{t+1} + I^l_{t+1}(D^l_{t+1} - R^l_{t+1})$$

where we have assumed that, in a neighborhood of the 1/2 cutoff, incumbents' decisions to retire are independent of the candidate quality differential (so the limit of $E[I_{it+1}(D_{it+1} - R_{it+1}) | v_{it} = \frac{1}{2} \pm \epsilon]$ as ϵ tends to zero we can be factorized as $I^k_{t+1}(D^k_{t+1} - R^k_{t+1})$, for $k = w, l$).

The RD estimand is therefore

$$\begin{aligned}\tau^{RD} &= v^w_{t+1} - v^l_{t+1} \\ &= (Par^w_{t+1} - Par^l_{t+1}) + \theta(I^w_{t+1} + I^l_{t+1}) + I^w_{t+1}(D^w_{t+1} - R^w_{t+1}) + I^l_{t+1}(R^l_{t+1} - D^l_{t+1})\end{aligned}$$

where

- $(P^w_{t+1} - P^l_{t+1})$: partisanship of barely-winner districts minus partisanship of barely-loser districts

[1] As in the paper, we use the term barely-winner to refer to districts where the Democratic party barely won the t election, and the term barely-loser to to refer to districts where the Democratic party barely lost the t election.

- $(D^w_{t+1} - R^w_{t+1})$: quality of Democratic candidate minus quality of Republican candidate in barely-winner districts

- $(R^l_{t+1} - D^l_{t+1})$: quality of Republican candidate minus quality of Democratic candidate in barely-loser districts

- $(I^w_{t+1} + I^l_{t+1})$: proportion incumbent-held seats in barely-winner districts plus proportion incumbent-held seats in barely-loser districts

We make the following simplifying assumptions:

- The RD condition $(Par^w_t - Par^l_t)$ of equal average Par between barely-winner and barely-loser districts continuous to hold at $t+1$, $(Par^w_{t+1} - Par^l_{t+1}) = 0$ (so change between t and $t+1$ in Par must affect barely-winner and barely-loser districts equally)

- The average quality differentials are equal in barely loser and barely winner districts, $(D^w_{t+1} - R^w_{t+1}) = (R^l_{t+1} - D^l_{t+1}) = QD$ (note that, for districts where an incumbent is running at $t+1$, $(D^w_{t+1} - R^w_{t+1})$ and $(R^l_{t+1} - D^l_{t+1})$ are, respectively, the incumbent-challenger quality differentials in barely-winner and barely-loser districts).

Under these assumptions, the RD estimand simplifies to:

$$\tau^{RD} = \theta \cdot (I^w_{t+1} + I^l_{t+1}) + QD \cdot (I^w_{t+1} + I^l_{t+1})$$

Thus, we can recover the personal incumbency advantage, $\theta + QD$, from the RD design as

$$\theta + QD = \frac{\tau^{RD}}{(I^w_{t+1} + I^l_{t+1})}$$

Note that, in the special case where no incumbent elected at t retires at $t+1$, we have $\theta + QD = \tau^{RD}/2$. But our model is more general and applies to contexts where the retirement rate is positive at $t+1$ (as long as retirement is non-strategic); as shown above, in this general case, the correction factor is $(I^w_{t+1} + I^l_{t+1})$, the proportion of incumbents who seek reelection in the treatment group plus the proportion of incumbents who seek reelection in the control group, which will be a number less than 2.

Note also that in this general model QD can be either positive or negative. On the one hand, incumbents may be of higher average quality than challengers due to a scare-off effect. But on the other hand, parties may recruit high-quality challengers at $t + 1$ to target seemingly vulnerable incumbents who barely survived at t. Researchers should make assumptions regarding the sign of QD that reflect the particular features of the electoral context they study.

In the main body of the paper, we restricted the analysis to open seats at t. The main reason for this restriction is that it makes plausible the assumption that freshman incumbents in both barely-winner and barely-loser districts are higher average quality than challengers, $QD > 0$, since it eliminates the possibility of strategic entry of candidates that is likely to occur at $t + 1$ in seats where incumbents barely survived or challengers barely succeeded. Another related reason is that the rate of retirement at $t + 1$ among freshman incumbents (who are first elected in open seats at t) will tend to be lower than among veteran incumbents in most electoral contexts. This will make the assumption of no strategic retirements much less restrictive—because the bounds on the effect will tend to be close to the effect in the observed sample when the rate of missingness is very low, that is, a few missing observations are unlikely to alter the conclusions of the study if the missing outcome has bounded support. Indeed, in our empirical illustration with U.S. House elections, there are almost no incumbent retirements at $t + 1$ in our open seat sample composed of freshman incumbents first elected at t.

3 Model in potential outcomes notation

We now write down the problem in terms of potential outcomes, and show the assumptions under which this potential outcomes framework leads to the model we outlined in the main body of the paper.

The term "potential outcomes" refers to all possible values of the Democratic vote share given all possible combinations of Democratic party winning or losing at election t and the incumbent elected at t running for reelection or retiring at $t+1$. The notation is as follows:

- v_{it+1} is the observed Democratic vote share at election $t+1$.
- $W_{it} = 1$ if Democratic party won election t , $W_{it} = 0$ if Democratic party lost election t.
- $I^R_{it+1} = 1$ if Republican incumbent running at $t+1$ election (decision made before election $t+1$ is held)
- $I^D_{it+1} = 1$ if Democratic incumbent running at $t+1$ election (decision made before election $t+1$ is held)
- There are six potential outcomes, written $v_{it+1}(i, j, k)$, where
  - i^{th} position indicates whether Democratic party won at t
  - j^{th} position indicates whether Democratic incumbent is running at $t+1$
  - k^{th} position indicates whether Republican incumbent is running at $t+1$

The six potential outcomes are illustrated in Table S1 below. Assuming there are no intervening elections between t and $t+1$, of these six potential outcomes, two are unfeasible: if the Democratic party wins at t, there can be either a Democratic incumbent candidate running at $t+1$ or no incumbent running, but there can be no Republican incumbent; similarly, if the Republican party wins at t, there can be either a Republican incumbent candidate running at $t+1$ or no incumbent running, but it is not possible to have a Democratic incumbent. This was also noted and incorporated into our model in the main body of the paper. The two unfeasible potential outcomes are illustrated in the shaded cells of Table S1.

Table S1: Potential outcomes

		Election $t+1$		
		Dem inc. runs	Rep inc runs	No incumbent runs
Election t	Dem party wins	$v_{it+1}(1,1,0)$	$v_{it+1}(1,0,1)$	$v_{it+1}(1,0,0)$
	Dem party loses	$v_{it+1}(0,1,0)$	$v_{it+1}(0,0,1)$	$v_{it+1}(0,0,0)$

Shaded cells indicate unfeasible outcomes.

Given these potential outcomes, the *observed* outcome, v_{it+1}, can be written as follows:

$$v_{it+1} = W_{it}\left\{v_{it+1}(1,1,0)\cdot I^D_{it+1} + v_{it+1}(1,0,1)\cdot I^R_{it+1} + v_{it+1}(1,0,0)\cdot(1-I^D_{it+1})\right\}$$
$$+(1-W_{it})\left\{v_{it+1}(0,1,0)\cdot I^D_{it+1} + v_{it+1}(0,0,1)\cdot I^R_{it+1} + v_{it+1}(0,0,0)\cdot(1-I^R_{it+1})\right\}$$

And, for the reasons mentioned above, $W_{it} = 1$ implies $I^R_{it+1} = 0$ and $W_{it} = 0$ implies $I^D_{it+1} = 0$, simplifying the expression to

$$\begin{aligned}
v_{it+1} = {} & W_{it}\left\{v_{it+1}(1,1,0)\cdot I^D_{it+1} + 0 + v_{it+1}(1,0,0)\cdot(1-I^D_{it+1})\right\} \\
& +(1-W_{it})\left\{0 + v_{it+1}(0,0,1)\cdot I^R_{it+1} + v_{it+1}(0,0,0)\cdot(1-I^R_{it+1})\right\} \\
= {} & W_{it}\left\{v_{it+1}(1,1,0)\cdot I^D_{it+1} + v_{it+1}(1,0,0)\cdot(1-I^D_{it+1})\right\} \\
& +(1-W_{it})\left\{v_{it+1}(0,0,1)\cdot I^R_{it+1} + v_{it+1}(0,0,0)\cdot(1-I^R_{it+1})\right\}
\end{aligned}$$

We impose the following structure

$$v_{it+1}(1,0,0) = v_{it+1}(0,0,0) \equiv Par_{it+1}$$

or that, in other words, the Democratic vote share is always the same in an open seat, regardless of whether the Democratic party won or lost the previous election. This is the same assumption we made in our model. In other words, we assume that the Democratic vote share is always the same in an open seat, regardless of whether the Democratic party won or lost the previous election. Moreover, Par is the baseline vote for the party in district i at $t+1$, given the district's partisanship, the election year's partisan trend, no incumbent candidate, and Democratic and Republican candidates of average quality.

We also assume that

$$v_{it+1}(1,1,0) = Par_{it+1} + \widetilde{v}_{it+1}(1,1,0)$$

$$v_{it+1}(0,0,1) = Par_{it+1} - \widetilde{v}_{it+1}(0,0,1)$$

and

$$\widetilde{v}_{it+1}(1,1,0) = \widetilde{v}_{it+1}(0,0,1) = \theta.$$

In other words, we assume that the Democratic party gains the same amount ($\theta > 0$) when a Democratic incumbent candidate is running, as it loses when a Republican incumbent is running. This is the same assumption made in our model in the main body of the paper, where we assumed that the incumbency advantage is the same for both parties, and equal to θ. Unlike in the paper, for simplicity, we do not consider candidate quality in this derivation, but we could add another term to the expression above so that $\widetilde{v}_{it+1}(1,1,0) = \widetilde{v}_{it+1}(0,0,1) = \theta + QD_{it+1}$, where QD_{it+1} is the quality differential.

The condition about "exogeneity" of retirement decisions is incorporated in the condition $v_{it+1}(1,1,0) = Par_{it+1} + \theta$ and $v_{it+1}(0,0,1) = Par_{it+1} - \theta$ (where $Par_{it+1} = v_{it+1}(0,0,0) = v_{it+1}(1,0,0)$ is Par, the baseline vote of the Democratic part in an open seat). The first condition says that the Democratic vote share that we see in a district where there is an open seat at $t+1$, is the same as the vote share that the Democratic party would have obtained in districts where the Democratic incumbent ran at $t+1$ if the incumbent had decided to retire instead. Analogously, the second condition says that the Democratic vote share that we see in a district where there is an open seat at $t+1$, is the same as the vote share that the Democratic party would have obtained in districts where the Republican incumbent ran at $t+1$ if the incumbent had decided to retire instead. In other words, open seats are valid counterfactuals for what the vote share of the Democratic party would have been in incumbent-held districts if these incumbents had decided to retire instead of actually running.

Imposing these conditions simplifies the expression for v_{it+1}:

$$\begin{aligned}
v_{it+1} =& W_{it}\left\{(Par_{it+1}+\theta)\cdot I^{D}_{it+1} + Par_{it+1}\cdot(1-I^{D}_{it+1})\right\} \\
&+(1-W_{it})\left\{(Par_{it+1}-\theta)\cdot I^{R}_{it+1} + Par_{it+1}\cdot(1-I^{R}_{it+1})\right\} \\
=& W_{it}\left\{Par_{it+1}\cdot I^{D}_{it+1} + \theta\cdot I^{D}_{it+1} + Par_{it+1} - Par_{it+1}I^{D}_{it+1}\right\} \\
&+(1-W_{it})\left\{Par_{it+1}\cdot I^{R}_{it+1} - \theta\cdot I^{R}_{it+1} + Par_{it+1} - Par_{it+1}I^{R}_{it+1})\right\} \\
=& W_{it}\left\{Par_{it+1} + \theta\cdot I^{D}_{it+1}\right\} + (1-W_{it})\left\{Par_{it+1} - \theta\cdot I^{R}_{it+1}\right\}
\end{aligned}$$

Now we take expectations

$$E(v_{it+1}|v_{it}=\frac{1}{2}+\epsilon) = E(Par_{it+1}|v_{it}=\frac{1}{2}+\epsilon) + E(\theta\cdot I^{D}_{it+1}|v_{it}=\frac{1}{2}+\epsilon)$$

$$E(v_{it+1}|v_{it}=\frac{1}{2}-\epsilon) = E(Par_{it+1}|v_{it}=\frac{1}{2}-\epsilon) - E(\theta\cdot I^{R}_{it+1}|v_{it}=\frac{1}{2}-\epsilon)$$

The RD estimand is the difference between the right and left limits of $E(v_{it+1}|v_{it}=v)$:

$$\begin{aligned}
\tau^{RD} &= \lim_{v\to\frac{1}{2}^{+}} E(v_{it+1}|v_{it}=v) - \lim_{v\to\frac{1}{2}^{-}} E(v_{it+1}|v_{it}=v) \\
&= \lim_{\epsilon\to 0} E(v_{it+1}|v_{it}=\frac{1}{2}+\epsilon) - \lim_{\epsilon\to 0} E(v_{it+1}|v_{it}=\frac{1}{2}-\epsilon)
\end{aligned}$$

Therefore,

$$\begin{aligned}\tau^{RD} =& \lim_{\epsilon \to 0}\left\{E(Par_{it+1}|v_{it} = \frac{1}{2} + \epsilon)\right\} - \lim_{\epsilon \to 0}\left\{E(Par_{it+1}|v_{it} = \frac{1}{2} - \epsilon)\right\} \\ &+\theta\left[\lim_{\epsilon \to 0}\left\{\cdot E(I^{D}_{it+1}|v_{it} = \frac{1}{2} + \epsilon)\right\} + \lim_{\epsilon \to 0}\left\{\cdot E(I^{R}_{it+1}|v_{it} = \frac{1}{2} - \epsilon)\right\}\right]\end{aligned}$$

Assuming continuity of Par_{it+1} (so that left and right limits coincide), we obtain:

$$\theta = \frac{\tau^{RD}}{\lim_{\epsilon \to 0}\left\{E(I^{D}_{it+1}|v_{it} = \frac{1}{2} + \epsilon)\right\} + \lim_{\epsilon \to 0}\left\{E(I^{R}_{it+1}|v_{it} = \frac{1}{2} - \epsilon)\right\}}$$

Alternative conditions

Alternatively, we could impose the more flexible structure:

$$v_{it+1}(1,1,0) = C_{it+1} + \widetilde{v}_{it+1}(1,1,0)$$

$$v_{it+1}(0,0,1) = C_{it+1} - \widetilde{v}_{it+1}(0,0,1)$$

where C_{it+1} is the baseline vote for the party in district i at $t+1$, given the district is held by an incumbent, the district's partisanship, the election year's partisan trend (we do not assume candidate quality plays a role to simplify the calculations, but if quality does play a role, we include it in C_{it+1}. This leads to

$$\begin{aligned}v_{it+1} =& W_{it}\left\{(C_{it+1} + \theta) \cdot I^{D}_{it+1} + Par_{it+1} \cdot (1 - I^{D}_{it+1})\right\} \\ &+(1 - W_{it})\left\{(C_{it+1} - \theta) \cdot I^{R}_{it+1} + Par_{it+1}(0,0,0) \cdot (1 - I^{R}_{it+1})\right\} \\ =& W_{it}\left\{C_{it+1} \cdot I^{D}_{it+1} + \theta \cdot I^{D}_{it+1} + Par_{it+1} \cdot (1 - I^{D}_{it+1})\right\} \\ &+(1 - W_{it})\left\{C_{it+1} \cdot I^{R}_{it+1} - \theta \cdot I^{R}_{it+1} + Par_{it+1}(0,0,0) \cdot (1 - I^{R}_{it+1})\right\}\end{aligned}$$

Again, we take expectations

$$E(v_{it+1}|v_{it} = \frac{1}{2} + \epsilon) = E(C_{it+1} \cdot I^D_{it+1}|v_{it} = \frac{1}{2} + \epsilon) + E(\theta \cdot I^D_{it+1}|v_{it} = \frac{1}{2} + \epsilon) + E(Par_{it+1} \cdot (1 - I^D_{it+1})|v_{it} = \frac{1}{2} + \epsilon)$$

$$E(v_{it+1}|v_{it} = \frac{1}{2} - \epsilon) = E(C_{it+1} \cdot I^R_{it+1}|v_{it} = \frac{1}{2} - \epsilon) - E(\theta \cdot I^R_{it+1}|v_{it} = \frac{1}{2} - \epsilon) + E(Par_{it+1} \cdot (1 - I^R_{it+1})|v_{it} = \frac{1}{2} - \epsilon)$$

The RD estimand is now:

$$\begin{aligned}
\tau^{RD} &= \lim_{v \to \frac{1}{2}^+} E(v_{it+1}|v_{it} = v) - \lim_{v \to \frac{1}{2}^-} E(v_{it+1}|v_{it} = v) \\
&= \lim_{\epsilon \to 0} E(v_{it+1}|v_{it} = \frac{1}{2} + \epsilon) - \lim_{\epsilon \to 0} E(v_{it+1}|v_{it} = \frac{1}{2} - \epsilon)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\tau^{RD} = &\lim_{\epsilon \to 0} \left\{ E(C_{it+1} \cdot I^D_{it+1}|v_{it} = \frac{1}{2} + \epsilon) \right\} - \lim_{\epsilon \to 0} \left\{ E(C_{it+1} \cdot I^R_{it+1}|v_{it} = \frac{1}{2} - \epsilon) \right\} \\
&+ \theta \left[\lim_{\epsilon \to 0} \left\{ \cdot E(I^D_{it+1}|v_{it} = \frac{1}{2} + \epsilon) \right\} + \lim_{\epsilon \to 0} \left\{ \cdot E(I^R_{it+1}|v_{it} = \frac{1}{2} - \epsilon) \right\} \right] \\
&+ \lim_{\epsilon \to 0} \left\{ E(Par_{it+1} \cdot (1 - I^D_{it+1})|v_{it} = \frac{1}{2} + \epsilon) \right\} - \lim_{\epsilon \to 0} \left\{ E(Par_{it+1} \cdot (1 - I^R_{it+1})|v_{it} = \frac{1}{2} - \epsilon) \right\}
\end{aligned}$$

Which is a more general expression and requires additional conditions to recover θ as above. In particular, a set of sufficient conditions is:

1. Local independence between retirement decisions and C_{it+1} and Par_{it+1}
2. The average decision to run for reelection after barely winning is the same for Democrats and Republicans, or, more precisely, equality of the limits w: $\lim_{v \to \frac{1}{2}^+} E(I^D_{it+1}|v_{it} = v) = \lim_{v \to \frac{1}{2}^+} E(I^R_{it+1}|v_{it} = v)$

These conditions lead, again, to

$$\tau^{RD} = \theta \cdot \left\{ \lim_{v \to \frac{1}{2}^+} \left\{ E(I^D_{it+1}|v_{it} = v) \right\} + \lim_{v \to \frac{1}{2}^-} \left\{ E(I^R_{it+1}|v_{it} = v) \right\} \right\}$$